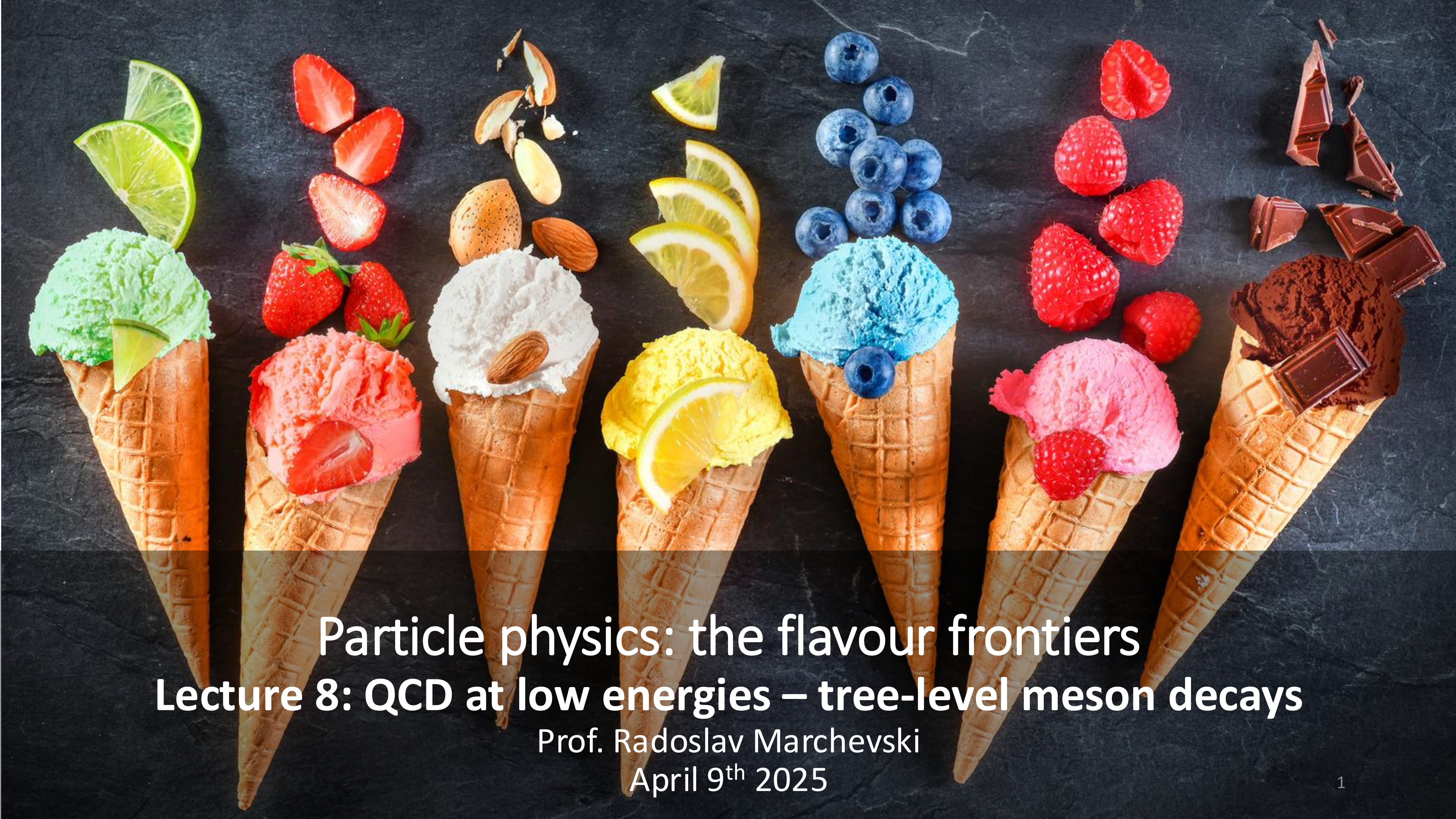


Particle physics: the flavour frontiers

Lecture 8: QCD at low energies – tree-level meson decays

Prof. Radoslav Marchevski

April 9th 2025

Short recap and today's learning targets

Last time we discussed

- What types of mesons we study in flavour physics experiments and how they are produced
- What are the most important experimental aspects of experimental flavour physics
- What are the main past and present facilities used to perform flavour physics experiments

Today you will …

- learn about some characteristics of QCD at low energy
- learn about the interplay between QCD and weak interactions in meson decays (factorization, decay constants, form factors, …)
- learn how to leptonic decays of charged mesons are described and which experimental techniques can be used to measure them

The Quark Model

- Simplest hadrons in the quark model:
  - *Mesons:* quark-antiquark constituents, $M = q\bar{q}$
  - *Baryons:* three quark constituents, $B = qqq$
  - *Antibaryons:* three antiquark constituents, $\bar{B} = \bar{q}\bar{q}\bar{q}$
- Quantum numbers of the hadrons are dictated by the quantum numbers of the constituent quarks and antiquarks
- Lightest mesons are the pions

$$\pi^+ = u\bar{d}, \qquad \pi^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right), \qquad \pi^- = \bar{u}d$$

- Lightest baryons are the proton and the neutron

$$p = uud, \qquad n = udd$$

- Mesons carry no baryon number, baryons carry a baryon number that we normalize to $B = +1$, and antibaryons carry a baryon number $B = -1$ (note that B can also be used to denote B-mesons)

Hadron masses

- The mass of the hydrogen atom comes from the sum of the electron mass, proton mass, and a potential (binding) energy ($V = -13.6$ eV)
- To make this statement, it is crucial that we are able to pull the electron and proton apart and measure their masses independent of their mutual electric field

- For hadrons it's much more complicated:
  - we can think of the hadron mass in a similar way to the mass of the hydrogen atom, sum of the mass of all constituent quarks and the binding energy that is of order $\Lambda_{\text{QCD}} \sim \mathcal{O}(300\ \text{MeV})$
  - but confinement implies that we can't separate quarks and measure their masses in isolation
  - how are we supposed to separate the contribution to the hadron masses from constituent quarks and QCD binding energy?

Hadron masses

- **How can we separate the contribution to the hadron masses from constituent quarks and QCD binding energy?**

- Option: measure the quark masses at high energy
  - couplings are small and can be treated perturbatively
  - the result is scale-dependent
  - effect of running of the couplings is very small for the lepton masses but dramatic for quark masses
  - we can't calculate the running quark mass below the confinement scale, $\mathcal{O}(\Lambda_{\mathrm{QCD}} \sim 300\ \mathrm{MeV})$
- We can split the quarks into three groups: light $(q = u, d, s)$, heavy quarks $(Q = c, b)$, and very heavy quark t
  - $q = u, d, s$: for light hadrons, the binding energy is the main contributor to the hadron mass
  - $Q = c, b$: for heavy hadrons (at least one heavy quark), the quark mass is the main contributor to the hadron mass and the binding energy is a correction of order $\mathcal{O}(\Lambda_{\mathrm{QCD}}/m_Q)$
  - t: decay width is very large $(\Gamma_t \gg \Lambda_{\mathrm{QCD}}) \Longrightarrow$ **to leading order the top quark is a free particle that doesn't form hadrons**

Hadron lifetimes

- We can classify hadrons as either *stable particles* or *unstable resonances*
  - stability is not meant in the absolute sense (the only truly stable hadron is the proton)
- **Two meanings of "stability":** *experiment-related* and *theory-related*

- **Experimentally,** we can determine the lifetime in two ways:
  - measure the decay width (by the energy of the decay products)
  - measure the distribution of the decay times
  - a particle is considered stable if its lifetime is large enough to be measured by the second method (or even larger so it escapes the detector before decaying)
- **Theoretically**, a particle is stable if it does not decay through QCD or QED but only via the weak interaction
  - hadrons whose decays violate the accidental flavour symmetry of QCD and QED
  - resonances are the particles that are not stable

Resonances

- Some resonances are so broad, with a width of the order of their mass
  - questionable if it's a bound state at all
  - in the PDG, particles whose name contains their mass in parenthesis $\left(\text{e.g. } \rho(770)\right)$ are resonances, the rest are particles
- **<u>Example of a resonance:</u>** ρ^+ meson
  - decays via QCD, almost always to two pions, $\rho^+ \rightarrow \pi^+\pi^0$
  - width is large $\Gamma_\rho \approx 150$ MeV just a factor of five less than its mass
- **<u>Example of a particle:</u>** π^+ meson
  - decays via the weak interaction, almost always via a W^+, $\pi^+ \rightarrow \mu^+\nu_\mu$
  - width is very small $\Gamma_\pi \approx 10^{-8}$ eV much smaller than its mass
- The **ratio $\boldsymbol{\Gamma_\rho/\Gamma_\pi \sim 10^{16}}$** demonstrates the difference between a resonance and a stable particle and explains why weakly decaying hadrons are called stable

Combining QCD with weak interactions

- Extracting experimentally the weak interaction parameters for quarks is not trivial
    - our Lagrangian is written in terms of quarks and gluons, while experimentally we observe hadrons
    - analysing physical processes in QFT we use asymptotically free states but quarks and gluons can't be free at infinity
    - we can deal with it by "parametrizing our ignorance"
- We isolate the parts of the amplitudes that are non-perturbative and apply the relevant methods to calculate them (or relate them to another measurable process)

- **Factorisation:** assume that we can factorise a process to a QCD part and a non-QCD part
- *Example:* purely leptonic decay $W^+ \to l\nu$ with an amplitude (matrix element)

$$\mathcal{A}_{W\to l\nu} = g\langle l\nu|\mathcal{O}|W\rangle, \qquad \mathcal{O}\sim\bar{l}_L\gamma_\mu W^\mu \nu_L$$

operator that mediates the process in the SM at tree level

- Simple calculation, the operator creates and annihilates the particles, and the amplitude is $\mathcal{A} \propto g$

Combining QCD with weak interactions

- *Example:* hadron decay $\pi^- \to l^- \bar{\nu}$ with an amplitude (matrix element)

$$\mathcal{A}_{\pi\to l\nu} = G_F V_{ud} \langle l\bar{\nu}|\mathcal{O}|\pi^-\rangle, \qquad \mathcal{O} \sim \bar{l}_L \gamma^\mu \nu_L \bar{u}_L \gamma_\mu d_L$$

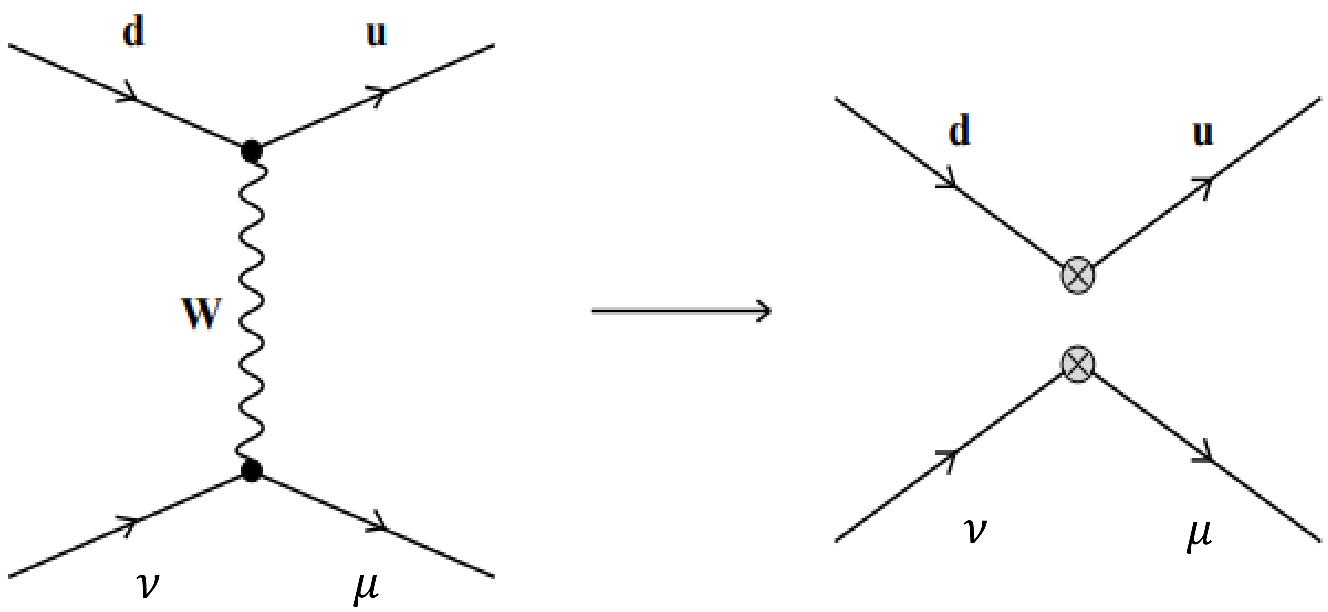
- the leptonic part is simple, the same as in the previous example

- *The pion is more complicated:* made of $u\bar{d}$ quarks in the quark model

- We can think of the $\bar{u}_L \gamma_\mu d_L$ part of the operator as the part that annihilates the pion
  - however, the $\bar{u}_L \gamma_\mu d_L$ operator annihilates free quarks, but they are not free inside the pion
  - factorisation hypothesis states that we can treat the leptonic and hadronic parts of the matrix element separately and write the results as a product of the two terms
  - leptons do not participate in the strong interaction so they can be factored out

$$\langle l\bar{\nu}|\mathcal{O}|\pi^-\rangle = \langle l\bar{\nu}|\mathcal{O}_l|0\rangle \times \langle 0|\mathcal{O}_H|\pi^-\rangle$$

Combining QCD with weak interactions



$$\langle l\bar{\nu}|\mathcal{O}|\pi^-\rangle = \langle l\bar{\nu}|\mathcal{O}_l|0\rangle \times \langle 0|\mathcal{O}_H|\pi^-\rangle$$

can be calculated perturbatively

can not be calculated perturbatively

Combining QCD with weak interactions: the decay constant

$$\langle 0|\mathcal{O}_H|\pi^-(p_\pi)\rangle \sim \langle 0|V^\mu - A^\mu|\pi^-(p_\pi)\rangle, \qquad V^\mu = \bar{u}\gamma^\mu d, \qquad A^\mu = \bar{u}\gamma^\mu\gamma_5 d$$

hadronic matrix element
(parametrized by form factors)

$$\mathcal{O}_H \sim \bar{u}_L\gamma_\mu d_L = \frac{1}{2}(V^\mu - A^\mu)$$

- The pion is pseudoscalar and the vacuum is parity-even → $\langle 0|V^\mu|\pi\rangle = 0$ and $\langle 0|A^\mu|\pi\rangle \propto p_\pi^\mu$
- We don't know how to calculate $\langle 0|A^\mu|\pi\rangle$ so we just define a proportionality constant f_π (Lorentz-scalar)

$$\langle 0|A^\mu|\pi\rangle \equiv -if_\pi p_\pi^\mu$$

- The decay constant f_π cannot be calculated but it can be extracted from measurements

$$\Gamma(\pi \to \mu\nu) = \frac{G_F^2|V_{ud}|^2}{8\pi} m_\mu^2 m_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 f_\pi^2$$

- From the measured decay rate we can extract: $f_\pi = 130.4 \pm 0.2$ MeV

Combining QCD with weak interactions: the decay constant

- Useful analog to gain physics intuition about the decay constant we can consider a perturbative system, the positronium e^+e^-
- We can define a positronium decay constant, which is calculable using QED
- Semi-classically the decay occurs when the electron and positron annihilate → they must be at the same place
- QM tell us that the decay amplitude is proportional to the wave function at $r = 0$ (distance between e^+ and e^-)
- This intuition can be carried over to QCD bound states
- The physics of the decay constant has to do with the wave function at the origin

$$f_\pi \propto |\Psi(r = 0)|$$

- We can't calculate it perturbatively, but we know some general scaling properties
- Lattice QCD provides precise nonperturbative calculations of decay constants

Approximate symmetries of QCD (two flavours only)

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} G_a^{\mu\nu} G_{a\,\mu\nu} + \sum_{q=u,d} \left(i\bar{q} D^\mu \gamma_\mu q - m_q \bar{q} q \right)$$

- In this Lagrangian, $m_u \neq m_d$ and $\mathcal{L}_{\text{QCD}}$ has a $[U(1)]^2$ flavour symmetry
- If we had $m_u = m_d$ the flavour symmetry would be $U(2) = SU(2)_I \times U(1)_B$ [isospin + baryon number symmetry]
- Under isospin the u and d quark form a doublet: $Q = \begin{pmatrix} u \\ d \end{pmatrix}$
- **Important:** isospin- $SU(2)_I$ is different than $SU(2)_L$ - u and d in the doublet have both LH and RH components
- All other quarks are singlets under $SU(2)_I$
- The up and down quarks are much lighter than Λ_{QCD}, $\mathcal{O}(300\text{ MeV})$, and isospin is an approximate symmetry of QCD, broken by a small parameter

$$\epsilon_I \equiv \frac{m_d - m_u}{\Lambda_{\text{QCD}}} \sim 10^{-3}$$

Approximate symmetries of QCD (two flavours only)

$$\epsilon_I \equiv \frac{m_d - m_u}{\Lambda_{\mathrm{QCD}}} \sim 10^{-3}$$

Why do we compare the quark masses with the QCD scale, Λ_{QCD} ?

Approximate symmetries of QCD (two flavours only)

$$\epsilon_I \equiv \frac{m_d - m_u}{\Lambda_{\text{QCD}}} \sim 10^{-3}$$

- Due to confinement, the energy of the system we're considering is at least of order of the QCD scale
- The symmetry-breaking parameter may be smaller, but it is not larger that ϵ_I
- In addition to ϵ_I the isospin symmetry is broken by QED due to the different charges of the u and d quarks
- The size of the breaking is of order $\alpha \sim 0.01$, similar to ϵ_I
- Approximate $SU(2)_I$ → hadron mass eigenstates can be assigned into well-defined representation of isospin
  - hadrons within an isospin multiplet are approximately degenerate – consequence of the symmetry of $\mathcal{L}_{\text{QCD}}$
- The degeneracy is indeed manifest in the baryon spectrum

$m_{p(uud)} = 938.272$ MeV, $m_{n(udd)} = 939.565$ MeV $\quad I = 1/2$ doublet

$\Delta^-(ddd)$, $\Delta^0(ddu)$, $\Delta^+(duu)$, $\Delta^{++}(uuu)$, $m_\Delta \sim 1230$ MeV $\quad I = 3/2$ quadruplet

Approximate symmetries of QCD (two flavours only)

$$\epsilon_I \equiv \frac{m_d - m_u}{\Lambda_{\text{QCD}}} \sim 10^{-3}$$

- Due to confinement, the energy of the system we're considering is at least of order of the QCD scale
- The symmetry-breaking parameter may be smaller, but it is not larger that ϵ_I
- In addition to ϵ_I the isospin symmetry is broken by QED due to the different charges of the u and d quarks
- The size of the breaking is of order $\alpha \sim 0.01$, similar to ϵ_I
- Approximate $SU(2)_I$ → hadron mass eigenstates can be assigned into well-defined representation of isospin
  - hadrons within an isospin multiplet are approximately degenerate – consequence of the symmetry of $\mathcal{L}_{\text{QCD}}$
- In the meson spectrum one can also see a quasi-degeneracy

$m_{\pi^+(u\bar{d})} = 139.57$ MeV, $m_{\pi^0(u\bar{u}-d\bar{d})} = 134.98$ MeV, $m_{\pi^-(\bar{u}d)} = 139.57$ MeV $I = 1$ triplet

$m_{\eta^0(u\bar{u}+d\bar{d})} = 547.86$ MeV $I = 0$ singlet

Much higher mass! Why?

Approximate symmetries of QCD (three flavours)

$$\eta^0 = \frac{u\bar{u} + d\bar{d}}{2}$$

- This assignment is only valid in the limit, where only the u and d quarks are light compared to Λ_{QCD}
- In the real world the s quark is also light, and the η^0 meson also includes an $s\bar{s}$ component

$$\boldsymbol{\eta^0 = \frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right)} \qquad m_{\eta^0} = 547.86\ \text{MeV}$$

- In addition to spectroscopy, isospin plays an important role in weak decays
  - relates form factors and even allows one to predict their values in specific kinematic points
- We can also treat the s quark light compared with Λ_{QCD}
- In this case the QCD Lagrangian gains an approximate $SU(3)_F$ flavour symmetry with u, d, s forming a triplet
- The breaking of this symmetry is much larger than isospin $\epsilon_I \sim (m_s - m_d)/\Lambda_{\text{QCD}} \sim 0.3$
  - large breaking but the symmetry considerations are still very useful (we won't discuss this further)

Approximate symmetries of QCD: heavy quark symmetry

- *Heavy quark symmetry applies to hadrons that contain one heavy quark*
- When the heavy quark mass goes to infinity the theory gains extra symmetries (not manifest in $\mathcal{L}_{\mathrm{QCD}}$) but rigorously defined and provides quite useful predictions

<u>Use QM of the Hydrogen atom as an example:</u>

- no big difference between the chemical properties of the hydrogen isotopes (hydrogen, deuterium, tritium)
- electrons are largely insensitive to the mass and the spin of the nucleus
- the light DoF are insensitive to the heavy DoF that source the potential: the light DoF determine the chemistry
- the potential is insensitive to the mass and spin of the nuclear source of the electromagnetic field (to zeroth order)
- Two leading corrections

difference in the reduced mass of the system

$$\mu \equiv \frac{m_e m_A}{m_e + m_A} \approx m_e \left(1 - \frac{m_e}{m_A}\right)$$

hyperfine splitting

$$\Delta E^{\mathrm{hf}} \sim m_e \alpha^4 \frac{m_e}{m_A}$$

Approximate symmetries of QCD: heavy quark symmetry

- Consider a meson that in the quark model is made of a b quark and a light u or d quark
- Analogously to the hydrogen atom we can think of the light DoF (u or d) like the electron, but binding comes from QCD instead of QED
  - we need to overcome the confining nature of QCD
  - for b quarks $m_b \gg \Lambda_{\text{QCD}}$ and we can get good results in the symmetry ($m_b \to \infty$) limit
- Let's consider specifically the $B(J=0)$ and $B^*(J=1)$ mesons, which in the quark model are the singlet and triplet states of the hyperfine structure
- In the $m_b/\Lambda_{\text{QCD}} \to \infty$ limit the B and B^* are degenerate (analog of the hyperfine splitting in the hydrogen system)

mass splitting

$$m_{B^*} - m_B \propto \frac{\text{const}}{m_{B^*} + m_B}$$

Approximate symmetries of QCD: heavy quark symmetry

$$m_{B^*} - m_B \propto \frac{\text{const}}{m_{B^*} + m_B}$$

- **<u>Prediction:</u>** for any heavy quark Q

$$\left(m_{Q^*} - m_Q\right)\left(m_{Q^*} + m_Q\right) = m_{Q^*}^2 - m_Q^2 = \text{const}$$

- The prediction is experimentally confirmed (small differences due to higher order effects)

$$m_{B^*}^2 - m_B^2 \approx 0.47 \text{ GeV}^2, \qquad m_{D^*}^2 - m_D^2 \approx 0.55 \text{ GeV}^2$$

- Similar in other mesons (D, D_s, B_s) and their exited states
- <u>*Feature of HQS:*</u> the wave function representing the internal B and D structure is independent on the color source
- *Example:* $B \to Dl\nu$ decay in the kinematic point where the two mesons are relatively at rest
  - the wave function of the light DoF is not affected by the $B \to D$ decay (up to a normalization) and we have

$$\langle B(p)|\mathcal{O}|D(p)\rangle = 1$$

Meson decays

- $M^{\pm}$ can **only decay** to a final state f (f is a final state of particles $a, b, \ldots$ with momenta $\vec{p}_a, \vec{p}_b, \ldots$)
- $M^0, \bar{M}^0$ can **decay** and/or **oscillate** into one another
- Decay amplitudes*

$$\underbrace{\mathcal{A}_f = \langle f|\mathcal{O}|M\rangle, \quad \bar{\mathcal{A}}_{\bar{f}} = \langle \bar{f}|\mathcal{O}|\bar{M}\rangle,}_{\text{charged and neutral}} \quad \underbrace{\bar{\mathcal{A}}_f = \langle f|\mathcal{O}|\bar{M}\rangle, \quad \mathcal{A}_{\bar{f}} = \langle \bar{f}|\mathcal{O}|M\rangle}_{\text{only neutral}}$$

M^0 and $\bar{M}^0$ can decay to a common final state

M^0 (or $\bar{M}^0$) can decay to CP- conjugate final state

- Theoretical calculation of decay amplitudes [low energy effective theories (EFT)]
  - **Short-distance (SD)** dynamics (e.g. W exchange)
  - QCD corrections (different techniques depending on the mass scale)
  - Hadronic matrix elements (non-perturbative dynamics at low energies)

} **Long-distance (LD)** dynamics

* $\bar{M}$ is the CP-conjugate of M $\left(M^0 \overset{CP}{\rightarrow} \bar{M}^0, M^+ \overset{CP}{\rightarrow} M^-\right)$; $\bar{f}$ is the CP-conjugate of f; i.e the state of $\bar{a}, \bar{b}, \ldots$ with $-\vec{p}_a, -\vec{p}_b, \ldots$

Meson decays

- **Observables:** $|\mathcal{A}_f|^2, |\bar{\mathcal{A}}_{\bar{f}}|^2, |\mathcal{A}_{\bar{f}}|^2, |\bar{\mathcal{A}}_f|^2$ or any functions of them

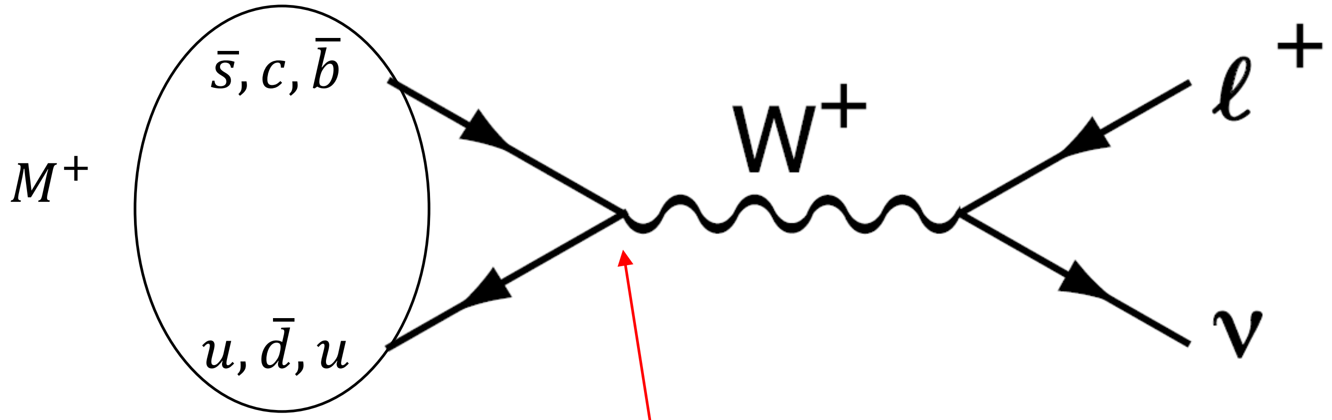
- **Decay rate:** (valid also for any CP-conjugate process)
  - $\Gamma(M \to f) \propto |\mathcal{A}_f|^2$
  - SD part of $|\mathcal{A}_f|^2$ is proportional to $|V_{ij}|^2$
  - $\Gamma(M \to f)$ allows extraction of $|V_{ij}|^2$
  - Link is complicated due to non-perturbative effects

Extraction of $|V_{ij}|$ from $\Gamma(M \to f)$ requires control of theoretical uncertainties

Theoretical uncertainties depend on the experimental technique to measure $\Gamma(M \to f)$

Charged mesons: leptonic decays

- Annihilation process (tree level)



$K^+ \rightarrow l^+\nu \; (l = e, \mu)$

$D^+, B^+ \rightarrow l^+\nu \; (l = e, \mu, \tau)$

$$\Gamma^{(0)}(M \rightarrow l\nu) = \frac{G_F^2}{8\pi} f_M^2 m_l^2 m_M \left(1 - \frac{m_l^2}{m_M^2}\right)^2 \left|V_{q_1 q_2}\right|^2$$

flavour structure (SD)

Phase space and helicity factor

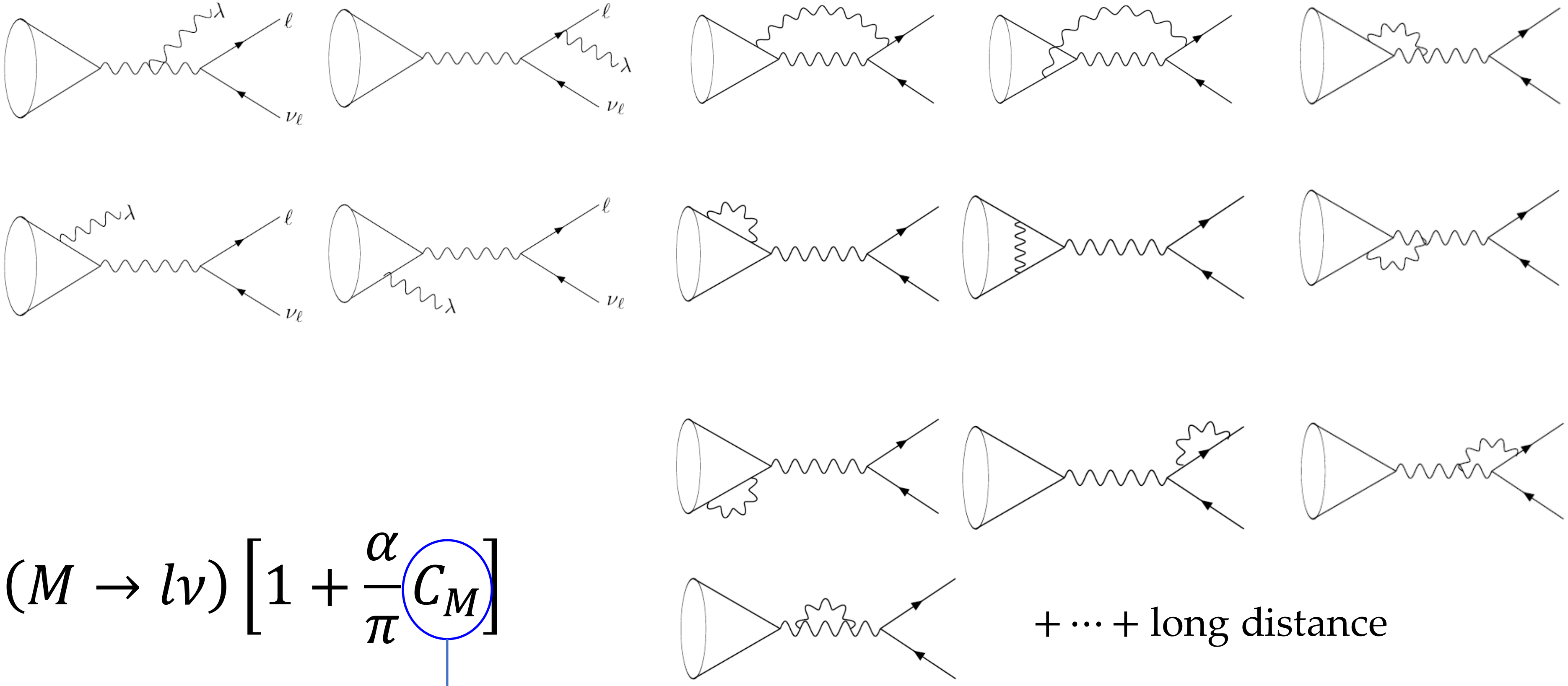
Decay constant: encodes the non-perturbative part of the hadronic matrix element ("quark wave function overlap")

Charged mesons: leptonic decays

- Annihilation process
  (radiative corrections)

$K^+ \to l^+\nu \ (l = e, \mu)$

$D^+, B^+ \to l^+\nu \ (l = e, \mu, \tau)$



$$\Gamma(M \to l\nu[\gamma]) = \Gamma^{(0)}(M \to l\nu)\left[1 + \frac{\alpha}{\pi} C_M\right]$$

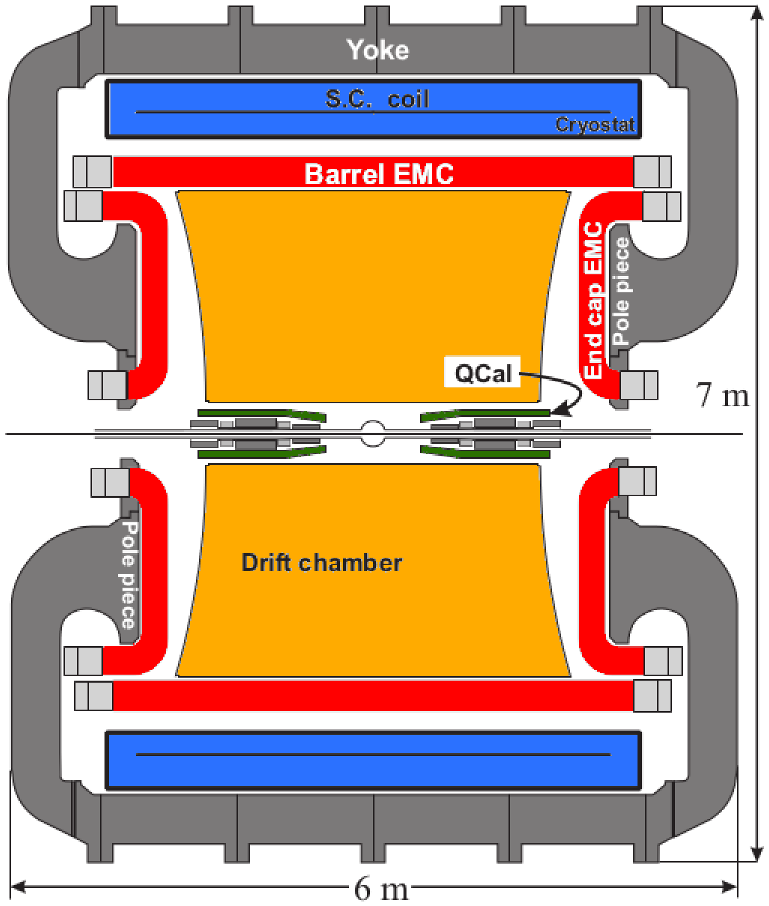
encodes the radiative connections (SD and LD)

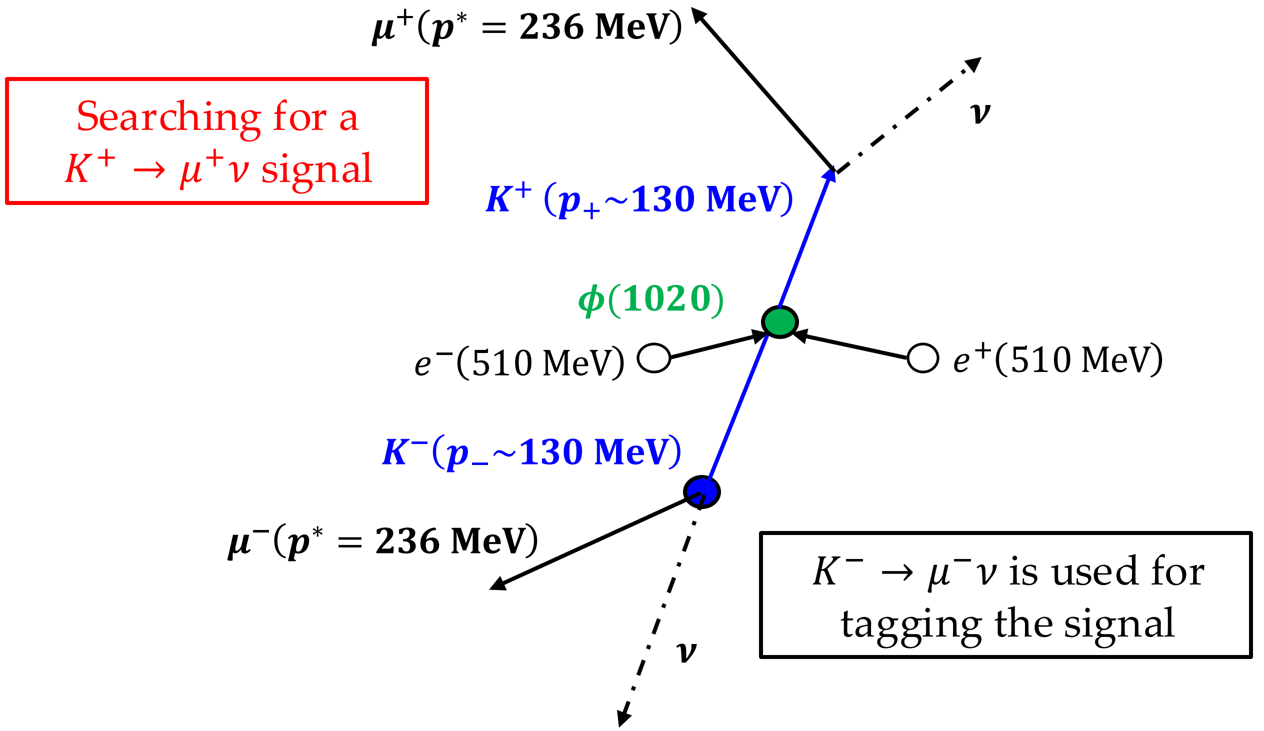
- The theoretical value to compare with measurement depends on the experimental treatment of the radiative **photons** in the final state: energy threshold, detection efficiency, etc. etc.

Charged mesons: $K^+ \rightarrow \mu^+\nu_\mu(\gamma)$ measurement

[Phys.Lett.B632:76-80(2006)]

- Latest measurement performed with the KLOE experiment at the ϕ-factory DAΦNE in Frascati
- Integrated luminosity of ~ 175 pb^{-1} collected in 2001-02 ($\sim 5.2 \times 10^8$) ϕ-meson decays
- e^+ (510 MeV) + e^-(510 MeV) collide at a 25 mrad angle → $\phi(1020)$ has a small transverse momentum (~12.5 MeV)



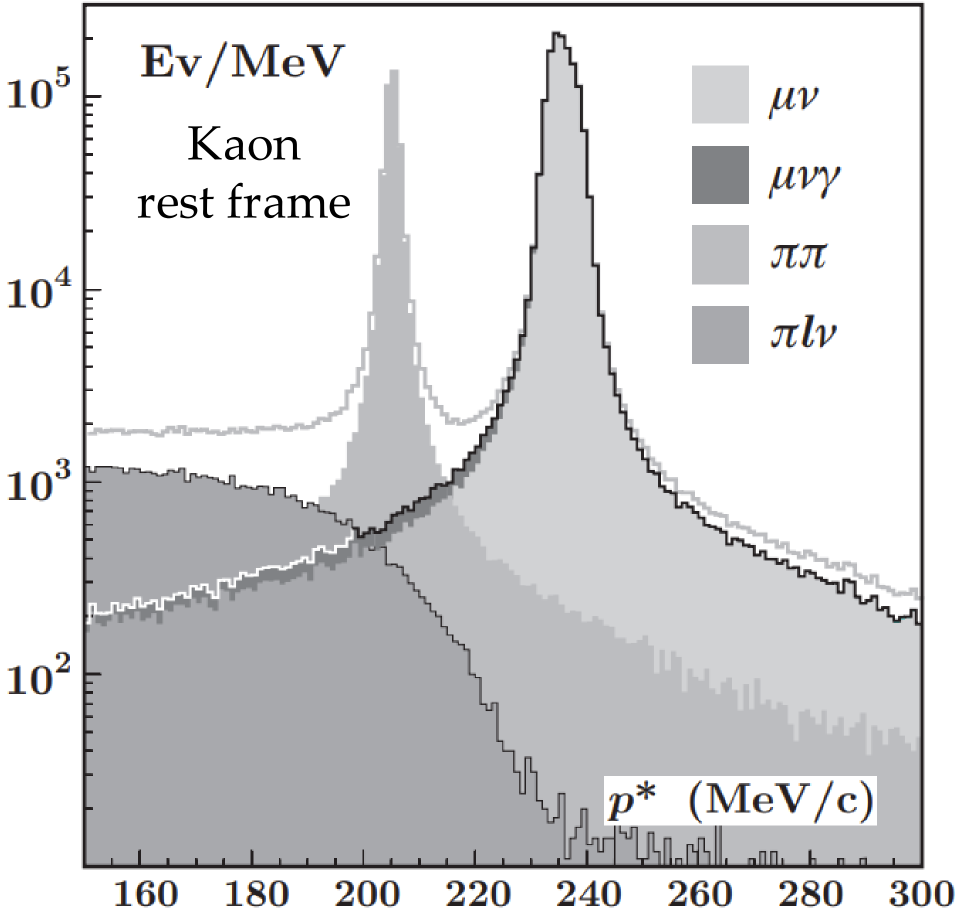


Charged mesons: $K^+ \to \mu^+ \nu_\mu(\gamma)$ measurement

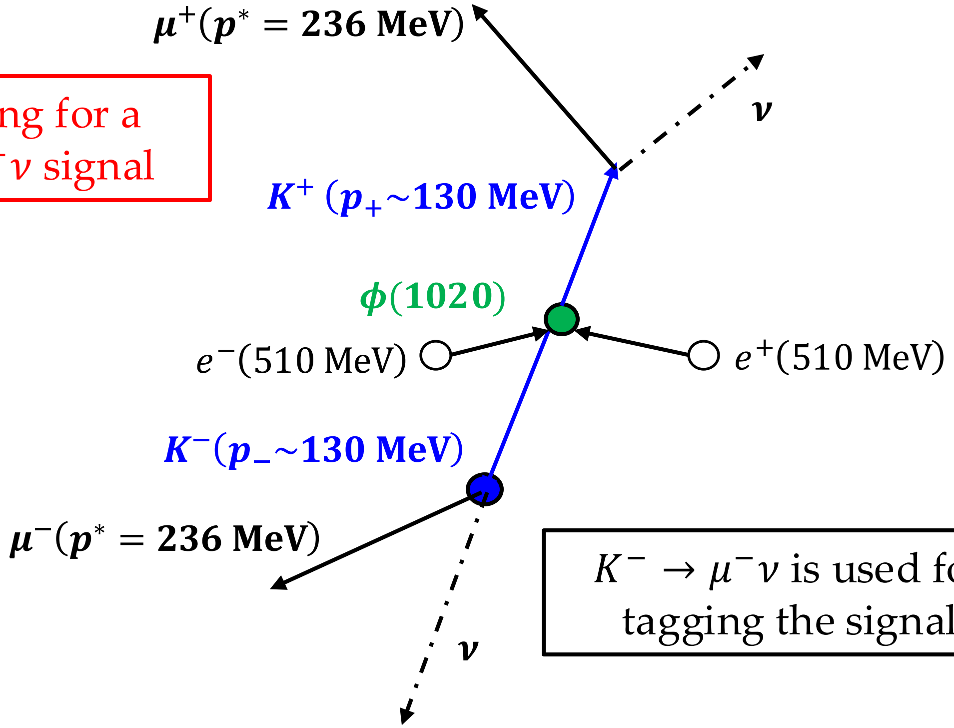
[Phys.Lett.B632:76-80(2006)]

$$BR(K^+ \to \mu^+ \nu(\gamma)) = \frac{N_{K^+ \to \mu^+ \nu(\gamma)}}{N_{Tag}} \times \frac{1}{\epsilon_{tot}}$$

signal efficiency



Searching for a $K^+ \to \mu^+ \nu$ signal



$K^- \to \mu^- \nu$ is used for tagging the signal

$$BR(K^+ \to \mu^+ \nu(\gamma)) = (63.66 \pm 0.09_{stat} \pm 0.15_{syst})\%$$

Charged mesons: $K^+ \to \mu^+ \nu_\mu(\gamma)$ measurement - $|V_{us}|$ extraction

[Phys.Lett.B632:76-80(2006)]

- One can determine $|V_{us}|$ using the ratio of the inclusive decays $K^+ \to \mu\nu_\mu(\gamma)/\pi^+ \to \mu\nu_\mu(\gamma)$
- Using input from lattice QCD for the LD effects f_K/f_π ratio and other experimental and theoretical inputs we can extract $|V_{us}|/|V_{ud}|$ using the formula

$$\frac{BR(K \to \mu\nu(\gamma))}{BR(\pi \to \mu\nu(\gamma))} = \frac{f_K^2}{f_\pi^2} \frac{m_K \left(1 - \frac{m_\mu^2}{m_K^2}\right)^2}{m_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2} \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{\left[1 + \frac{\alpha}{\pi} C_K\right]}{\left[1 + \frac{\alpha}{\pi} C_\pi\right]}$$

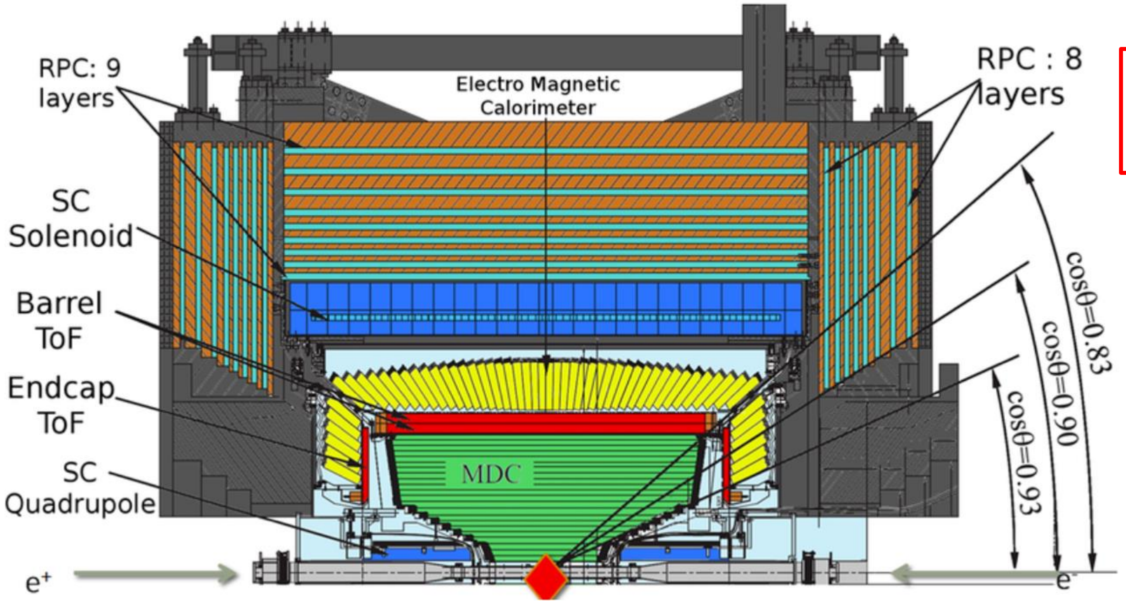
$$\frac{|V_{us}|^2}{|V_{ud}|^2} = 0.05211 \pm 0.00016_{\mathrm{exp}} \pm 0.00019_{\mathrm{radiative}} \pm 0.00117_{\mathrm{lattice}}$$

$$|V_{us}|_{K^+ \to \mu^+ \nu(\gamma)} = 0.2223 \pm 0.0026 \quad \text{(result from the paper)}$$

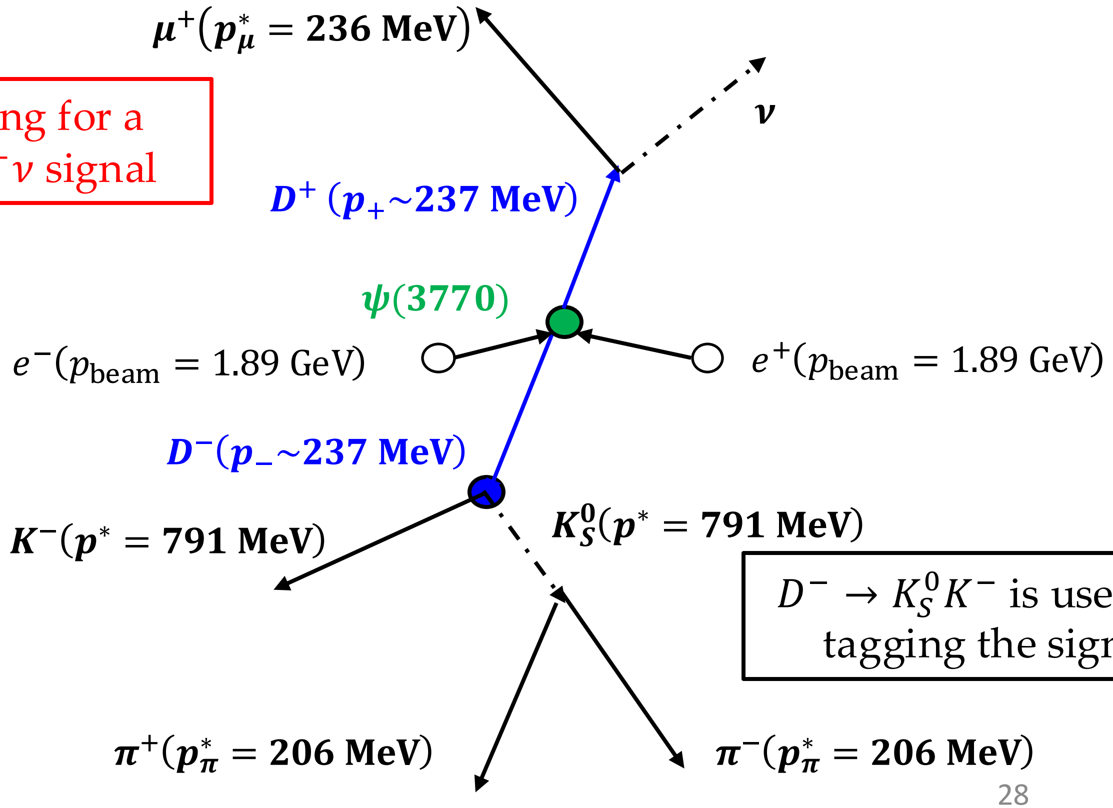
Charged mesons: $D^+ \to \mu^+\nu_\mu$ measurement

[Phys.Rev.D 89 (2014) 5, 051104]

- Latest measurement: BES III, integrated luminosity of 2.92 fb^{-1}, collected at $\sqrt{s} = 3.773$ GeV ($\psi(3770)$ resonance)
- $\psi(3770) \to D^+D^- \sim 41\%$ of the time
- Tagged D^- mesons are identified by their beam-energy-constrained mass $M_{BC} = \sqrt{E_{\text{beam}}^2 - |p_{K\pi\pi}|^2}$



Searching for a
$D^+ \to \mu^+\nu$ signal



$D^- \to K^0_S K^-$ is used for tagging the signal

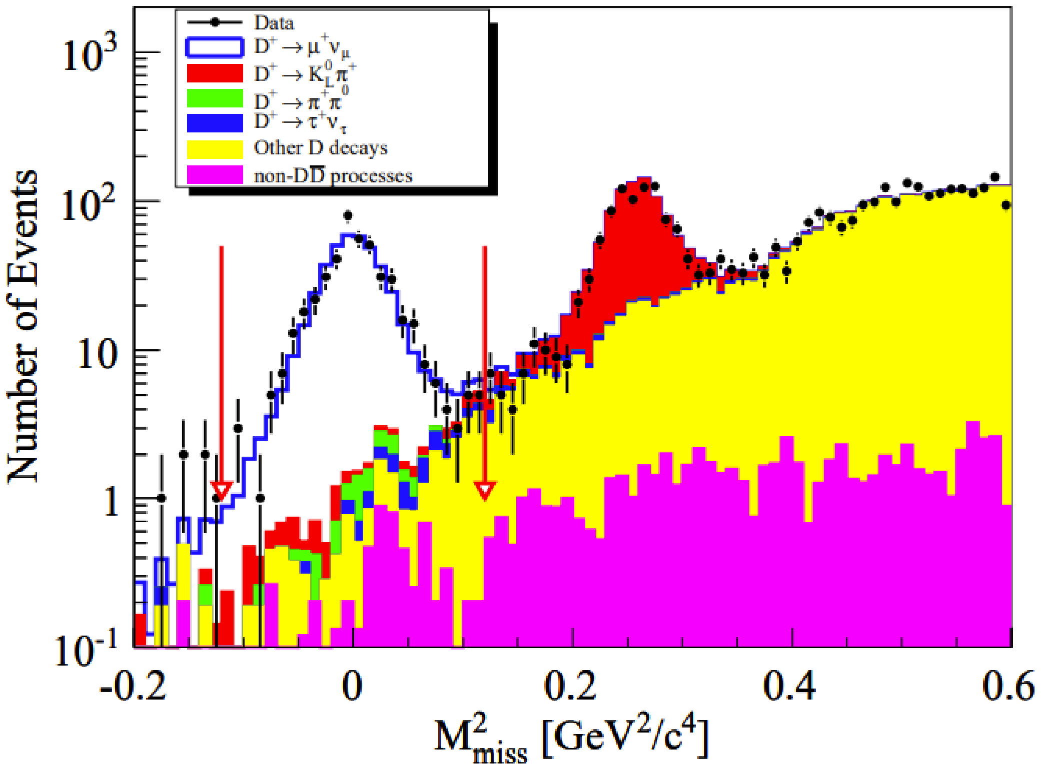
Charged mesons: $D^+ \to \mu^+\nu_\mu$ measurement

[Phys.Rev.D 89 (2014) 5, 051104]

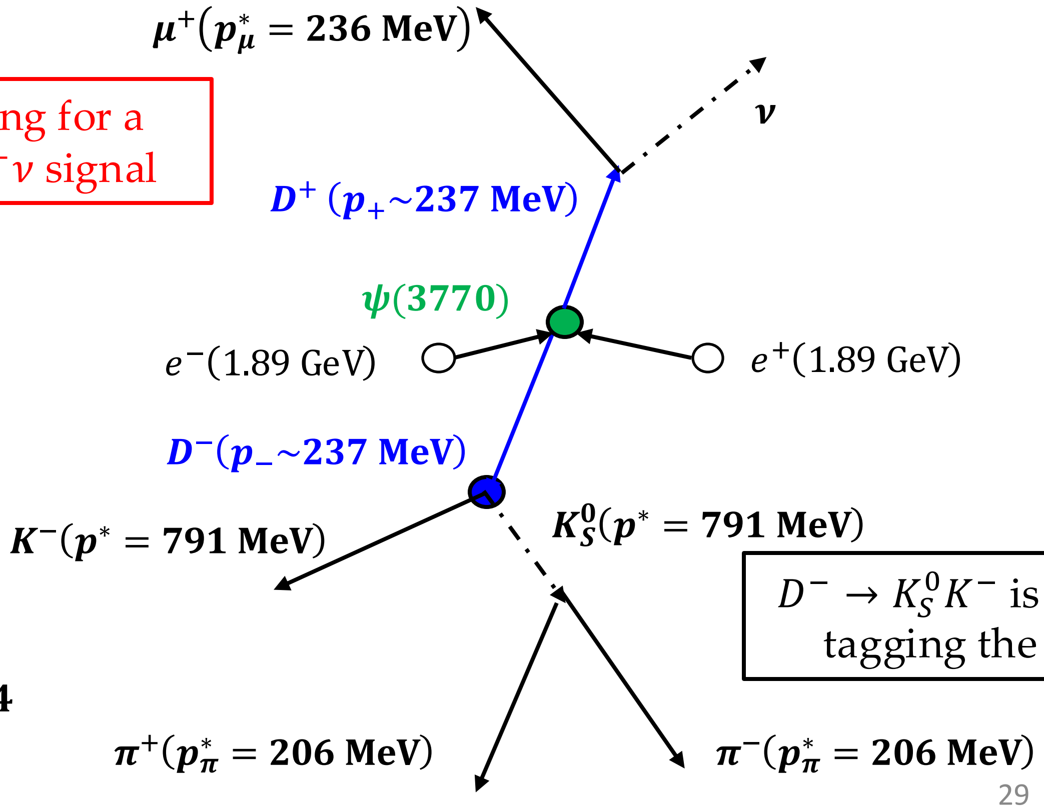
- Main kinematic variable used to define the region where we will look for signal: M^2_{miss}

- $M^2_{\text{miss}} = (E_{\text{beam}} - E_{\mu^+})^2 - (-p_{D^-_{\text{tag}}} - p^+_\mu)^2$

$p_{D^-_{\text{tag}}}$: 3-momentum of the tagged D^- candidate



Searching for a $D^+ \to \mu^+\nu$ signal



$D^- \to K^0_S K^-$ is used for tagging the signal

$$BR(D^+ \to \mu^+\nu) = (3.71 \pm 0.19_{\text{stat}} \pm 0.06_{\text{syst}}) \times 10^{-4}$$

Charged mesons: $D^+ \to \mu^+\nu_\mu$ measurement - $|V_{cd}|$ extraction

[Phys.Lett.B632:76-80(2006)]

- One can determine $|V_{cd}|$ using the master formula for the branching ratio

$$\Gamma(D^+ \to \mu^+\nu) = \frac{G_F^2}{8\pi} f_{D^+}^2 m_\mu^2 m_{D^+} \left(1 - \frac{m_\mu^2}{m_{D^+}^2}\right)^2 |V_{cd}|^2$$

- Using the measured branching fraction, G_F, the muon and D^+ mass, and the lifetime of the D^+ meson we get:

$$f_{D^+}|V_{cd}| = \left(45.75 \pm 1.20_\mathrm{stat} \pm 0.39_\mathrm{syst}\right) \mathrm{MeV}$$

- f_{D^+} is obtained using as input the CKM matrix element $|V_{cd}| = 0.22520(65)$ from the global SM fit

$$f_{D^+} = \left(203.2 \pm 5.3_\mathrm{stat} \pm 1.8_\mathrm{syst}\right) \mathrm{MeV}$$

- $|V_{cd}|$ is determined using $f_{D^+} = (207 \pm 4)$ MeV from lattice QCD as input

$$|V_{cd}| = 0.2210 \pm 0.0058_\mathrm{stat} \pm 0.0047_\mathrm{syst}$$

Summary of Lecture 8

<u>Main learning outcomes</u>

- What are some of the main characteristics of QCD at low energy
- What is the interplay between QCD and weak interactions in meson decays (factorization, decay constants, form factors, …)
- How to describe theoretically leptonic decays of charged mesons and which experimental techniques can be used to measure them